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Drift-Free Magnetic Geometries in Adiabatic Motion

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Abstract

There exist magnetic fields in which particles bouncing between mirror points experience no net first-order guiding center drift. In such fields, even though the instantaneous gradient and curvature drifts are not zero, their total effect integrated over any bounce period vanishes, so that particles merely wobble back and forth around fixed field lines. A class of two-dimensional drift-free fields, somewhat resembling the configuration found in the geomagnetic tail, is described; several proofs of the drift-free property are given, including some that suggests that the property of vanishing net drift might extend to non-adiabatic orbits. A general criterion for identifying drift-free fields is developed and a case of motion in a nearly drift-free field is also investigated. The theory is applied to the plasma sheet in the earth's magnetotail and observational evidence is presented suggesting that the magnetic field there indeed approaches a drift-free configuration.

Introduction

The purpose of this work is to examine a certain class of charged particle motions in a magnetic field. The motions involve "magnetic mirror" geometries in which two independent periodic modes are possible - gyration around field lines and "bounce" between mirror points - with two associated adiabatic invariants μ and J. The fields discussed here have the additional property that the net guiding center drift, averaged over any bounce period, vanishes; as will be seen, an analogous property can also exist for non-adiabatic orbits.

An example of such a configuration is given in the next section: it has a finite curl and therefore it is not certain that particle distributions can be found which would provide the required current density, in the absence of any net charge transport. However, self-consistent plasma distribution functions certainly will exist for fields which are not drift-free but merely approach a drift-free configuration: in such cases the integrated drift does not vanish but is merely small, so that the field can carry a substantial plasma density with only a modest current density. An analogous situation exists in dynamo theory where Cowling [1934] proved the impossibility of axisymmetrical dynamos, yet Braginskii [1964 a, b] derived useful models by assuming a slight deviation from axial symmetry.

A magnetic configuration which can be treated as a perturbed version of a drift-free geometry apparently exists in the plasma sheet of the earth's magnetotail, explaining why the total current carried by this sheet is relatively small, considering the amount of plasma in the sheet. The outer magnetosphere of Jupiter, as observed by Pioneer 10 [Smith et al., 1974] can also be represented locally by a two-dimensional drift-free field, to which a small component is added to preserve axial near-symmetry, so that it, too, qualifies as a nearly drift-free configuration. The field in Jupiter's outer magnetosphere might also be approximating an axisymmetric drift-free geometry but, although such geometries are investigated in what follows, it is not known whether they actually exist.

A Simple Example

Consider a two-dimensional magnetic field of a form which is frequently used in models of the magnetotail plasma sheet

$$\underline{B} = B_x(z)\,\hat{\underline{x}} + B_z\,\hat{\underline{z}} \tag{1}$$

where

$$B_z = \text{constant} \tag{2}$$

Field lines of this configuration, in any plane of constant y and for the case where $B_x = \lambda z$, are given in Figure (1), which also depicts (schematically) the projection of the orbit of a particle undergoing guiding center motion. One may now ask, does such an orbit undergo a slow drift in the y direction, orthogonally to the plane of the figure?

This problem is easily handled by the mathematical device described below, although more direct proofs also exist and will be presented.

Suppose that an electric field

$$\underline{E} = E_0 \hat{\underline{y}} \tag{3}$$

exists across the magnetic field. (An electric field probably does exist across the plasma sheet, so that if equation (1) is assumed to represent a model of the magnetotail, equation (3) should probably also be assumed; this, however, is not relevant to the present calculation, in which the electric field is merely a mathematical device.) Let v_y be the bounce-averaged magnetic drift velocity of the given particle in the y direction and let q be its electric charge; the particle then will increase its kinetic energy W at the rate $q E_0 v_y$ per unit time.

However, if the problem is transformed into a frame moving with the uniform velocity

$$\underline{u} = \hat{\underline{x}} (E_0/B_z) \tag{4}$$

then in the non-relativistic approximation the electric field vanishes. If v_y remains unchanged by the transformation (see below) then in the moving frame no energy gain is possible: this can only be reconciled with the previous assumption if $v_y = 0$, i.e. the particle's guiding center "wobbles" around the guiding field line with no net drift.

To see whether v_y is conserved one should note that to the lowest relativistic order the magnetic field is not changed by the transformation to the moving frame and therefore any portions of v_y which depend on

$\underline{B}$ alone will also transform unchanged. In addition v_y will contain some terms involving the electric field [Northrop, 1963, eq. 1.17]. The above argument then states that $v_y = 0$ for any finite E_0 and it is plausible to assume (although this is not a rigorous proof) that the same also holds in the limit $E_0 = 0$, in which case the magnetic field is indeed drift-free.

If this approach is accepted one obtains two additional insights into this type of motion. First, there is no need to assume that the motion is adiabatic: the same conclusions should also be valid for non-adiabatic motion, as long as v_y is suitably defined. An exception occurs in the limit $B_z = 0$, when (4) can no longer be used; indeed, in this limit meandering modes of motion exist [Sonnerup, 1971] which have finite v_y.

Nearly Drift-Free Fields

A second advantage of the above approach is that it allows one to estimate v_y in some cases in which the geometry is only approximately drift-free.

Suppose B_z is not constant but varies slowly with x; then the preceding argument no longer holds, since $\underline{u}$ is no longer a uniform velocity. Assume now

$$B_x(0) = 0 \tag{5}$$

(as in the field in Fig. 1) and consider the adiabatic motion of a charged particle in the plane $z = 0$. The electric drift will carry such a particle in the x direction and due to conservation of the magnetic moment μ the particle's kinetic energy W will change with x at the rate

$$\partial W/\partial x = \mu \, \partial B_z/\partial x \tag{6}$$

By previous arguments the rate with which W changes in time equals $q E_0 v_y$ and it can also be found by multiplying (6) with the electric drift velocity E_0/B_z, giving

$$v_y = (\mu/qB_z) \, \partial B_z/\partial x \tag{7}$$

This can also be written as

$$v_y = \frac{1}{2} v R_g / L_x \tag{8}$$

where v is the particle's velocity, R_g its gyration radius and L_x the scale length of the variation of B_z. By way of comparison, the instantaneous guiding center drift velocity is of the order of $v R_g/L$, where L is the much smaller scale of variation of the total field.

The derivation of v_y for adiabatic particles not confined to the equatorial plane is much more difficult and requires the bounce-averaged Hamiltonian $K(\alpha, \beta, \mu, J)$, discussed later on.

Alternative Proofs

Consider Newton's equations of motion for a particle of charge q and mass m in the field of (1). The x component of the acceleration is

$$\ddot{x} = q B_z \dot{y} / m \tag{9}$$

and it integrates to

$$\dot{x} = q B_z (y - y_0)/m \tag{10}$$

If the motion in x is periodic, as happens in the adiabatic case, y will also vary in a periodic fashion. Even if the motion is non-adiabatic but x varies in a periodic or almost periodic fashion, the same property will hold true for y and no net drift will occur, showing that the motion is drift-free. As before, the argument cannot be used in the limit $B_z = 0$, since $\ddot{x}$ vanishes in that case, and as has already been noted, the motion in that limit is indeed not drift-free.

A more conventional proof of the drift-free character of adiabatic motion in the field (1) involves the averaging of the instantaneous guiding center drift over a bounce period. The calculation is lengthy and is therefore given in the appendix; it is assumed there that the magnetic configuration possesses mirror symmetry with respect to the plane $z = 0$ but, as is shown in what follows, this requirement is not essential.

The Averaged Hamiltonian

A powerful tool for treating motions in a magnetic mirror geometry is provided by the bounce-averaged Hamiltonian $K(\alpha, \beta, \mu, J)$, introduced by Northrop and Teller [1960]. Its use represents a second averaging, over the periodicity of the mirroring motion, in addition to the gyration averaging which leads to the guiding center theory.

Let (α, β) be Euler potentials of the magnetic field (e.g. review by Stern [1970]) and let a particle move in this field and conserve the two lowest adiabatic invariants μ and J. In the absence of electric fields the Hamiltonian $K(\alpha, \beta, \mu, J)$ for the averaged motion then equals the particle's kinetic energy W.

Attention will be confined to cases in which, due to some symmetry, K does not depend on β - in particular, either to 2-dimensional fields where

$$\underline{B} = \underline{B}(x, z) \quad ; \quad B_y = 0 \quad ; \quad \beta = y$$

or to axially symmetric fields where (using cylindrical coordinates ρ, z and φ)

$$\underline{B} = \underline{B}(\rho, z) \quad ; \quad B_\varphi = 0 \quad ; \quad \beta = \varphi$$

The drift velocity (combining gradient and curvature drifts) then parallels the direction of $\nabla\beta$ and its average value can be found by one of Hamilton's equations of motion

$$\langle\dot{\beta}\rangle = (c/e)\, \partial K/\partial\alpha \tag{11}$$

In particular, if K is independent of both α and β, the average drift in the direction of $\nabla\beta$ vanishes and the geometry will be drift-free. This is a general condition for drift-free geometries but it does constitute a useful criterion for identifying such fields, since the explicit form of K is difficult to derive [e.g. Chen and Stern, 1975]. We therefore proceed to develop an alternative criterion.

The Euler potentials are related to $\underline{B}$ through

$$\underline{B} = \nabla\alpha \times \nabla\beta \tag{12}$$

Let it be first assumed (an assumption which will later be removed) that α has mirror symmetry with respect to the plane $z = 0$, i.e. for a 2-dimensional field

$$\alpha(x, z) = \alpha(x, -z) \tag{13-a}$$

and for an axisymmetric field

$$\alpha(\rho, z) = \alpha(\rho, -z) \tag{13-b}$$

If a particle guided by the field line (α, β) of this field has kinetic energy W and adiabatic invariants (μ, J) and it is reflected at $z = z_m$, then

$$J = (4/m)\,(2W)^{1/2} \int_0^{z_m} (1 - B/B_m)^{1/2}\,(\partial s/\partial z)\Big|_\alpha\, dz \tag{14}$$

where $B_m = B(z_m, \alpha)$. The last equation implies that a function $f(z_m, \alpha)$ exists such that

$$J\,W^{-1/2} = f(z_m, \alpha) \tag{15}$$

The magnetic moment μ, on the other hand, satisfies

$$W/\mu \;=\; B_m \;=\; B(z_m, \alpha) \tag{16}$$

Elimination of z_m between the last two equations implicitely gives the functional form of $W(\alpha, \mu, J)$ which, in the absence of electric fields, equals the Hamiltonian K of (11). If W is independent of α, this means that such an elimination removes α as well, which will only happen if the right hand expressions depend on each other, i.e.

$$f(z_m, \alpha) \;=\; F(B(z_m, \alpha)) \tag{17}$$

This is easier to express if (14) is transformed to a form in which the field intensity B becomes the integration variable, namely

$$I(\alpha, B_m) \;=\; \int_{B_{eq}(\alpha)}^{B_m} (1 - B/B_m)^{1/2} \left.\partial s/\partial B\right|_{\alpha} dB \;=\; G(B_m) \tag{18}$$

Here $B_{eq} = B(z=0, \alpha)$ is the equatorial field intensity on the field line with the given value of α and the last equality, involving some function G independent of α, must hold if the field is to be drift-free.

A simple extension of the arguments of the section dealing with nearly drift-free fields shows that B_{eq} may not depend on α because if such a dependence existed, then in the limit $J \longrightarrow 0$ a particle would experience a net drift. In that limit a particle is confined to the plane $z = 0$: assuming an electric field

$$\underline{E} \;=\; E_0 \nabla \beta \tag{19}$$

it is found that the electric drift of such a particle would be in the direction of $\nabla\alpha$. If B_{eq} were allowed to vary with α then the particle's energy would change in the course of its electric drift due to the conversation of μ and, as outlined earlier, this would imply a net magnetic drift in the direction of $\nabla\beta$.

The condition that I is independent of α thus reduces to the requirement that $\partial s/\partial B$ at constant α depends on B alone - i.e.

$$s = s(B) \tag{20}$$

Another way of expressing this relationship would be in the form of a Jacobian

$$\partial(s, B)/\partial(q_1, q_2) = 0 \tag{21}$$

where (q_1, q_2) equals on (x, z) or (ρ, z) depending on the geometry. In the cases discussed here both s and B can be expressed in terms of the Euler potential α and its derivatives and (21) then reduces to a rather complicated integro-differential equation.

In all this the requirement of symmetry expressed by equations (13) can be waived, provided (5) holds. The reasoning behind this is outlined below: it can be readily expressed in conventional mathematical terms but doing so has little real effect beyond lengthening the presentation.

Suppose that the symmetry condition holds and consider two points $P_1 = (x, z)$ and $P_2 = (x, -z)$ in a two-dimensional field, symmetrically located on the same field line at equal distances from the plane $z = 0$

(for axial symmetry replace x everywhere by ρ). One now readily finds (e.g. from equations (A-1) and (A-2) in the appendix) that both the curvature drift and the gradient drift at P_1 and at P_2 are the same. By extension it can also be shown that the average drift of either kind experienced by the particle along the portion of its guiding field line above $z = 0$ is the same as the corresponding average drift below $z = 0$. Since the average gradient drift in any period cancels the average curvature drift, it also follows that a similar cancellation holds just for the portion of the orbit above $z = 0$ and just for the portion below $z = 0$. Thus, if one takes two different field line shapes satisfying (1), (2) and (5), and splices them together at $z = 0$, the motion in the resulting field will still be drift-free, as has been claimed.

In the case of the field of (1) and (2) it is possible to choose

$$\alpha = x B_z - \int_0^z B_x \, dz \qquad (22)$$

$$\beta = y \qquad (23)$$

and calculation then shows that both B and s depend on z alone, the elimination of which leads to the condition (20). Analytic examples or even existence proofs for the axially symmetric case are not known.

Application to the Plasma Sheet

The form of the magnetic field given in equation (1) and in particular the configuration shown in Figure 1 resemble in many way the magnetic field observed in the earth's tail, in the plasma sheet region. In that region the value of B_z gradually decreases from about 4γ at $20\ R_e$ to $0.5\ \gamma$ at $70\ R_e$ [Behannon, 1970, table on p. 751 ; Bowling and Wolf, 1974] , while $\partial B_x/\partial z \approx 4\ \gamma/R_e$ at a distance of $20\ R_e$.

The preceding theory suggests that the mean velocity v_y of charged particles across the plasma sheet is relatively small. If the current density observed in the tail can be entirely attributed to the mean drift of particles - a point examined separately later on - then the magnitude of the tail current can be used to estimate v_y in the following way.

At a distance of $20\ R_e$ the change in B_x across the plasma sheet amounts to about $20\ \gamma$, representing a current of about 10^5 amperes for each R_e of plasma sheet length. Assuming that the current is carried mainly by protons (which are considerably more energetic than electrons and therefore drift more rapidly) and taking the plasma density in the middle of the sheet as 0.5 protons/cm^3 over a width of $6\ R_e$, one finds that in order to maintain the observed current, the protons must move with an average cross-sheet velocity

$$v_y \approx 5.13 \text{ km/sec}$$

A very rough theoretical estimate of what v_y ought to be is obtained from eq. (8) , which assumes protons to be adiabatic and equatorial. If a typical plasma sheet proton has an energy of 2 kev, one finds $v \approx 620$ km/sec, $R_g \approx 1/3\ R_e$ (in a field of 3γ) while the work of Behannon [1970] suggests $L_x \approx 15\ R_e$, leading to

$$v_y \approx 6.9 \text{ km/sec}$$

The agreement obtained here should only be considered as qualitative, for at least two reasons. First, the conditions for which (8) has been derived are not generally met and secondly, we have ignored the effects due to density variations in a diamagnetic plasma.

Concerning the second point it should be noted that the plasma sheet current density $\underline{j}$ is contributed not only by the drift velocity v_y of charged particles but also by the variation in the density $\underline{M}$ of magnetic moment. The latter contribution is called the magnetization current density [e.g. Longmire, 1963] and equals $\nabla \times \underline{M}$.

In a strictly drift-free configuration of the type discussed here, if the plasma density N does not vary with x then $\nabla \times \underline{M}$ does not contribute to the total current, for the following reason. Assume that the plasma sheet is bounded at $z = \pm z_0$ by plasma-free regions (Figure 2). If $\nabla \times \underline{M}$ is now integrated over a rectangular cross-sectional area of the sheet, bounded by $z = \pm z_1$ $(z_1 > z_0)$, $x = x_1$ and $x = x_2$, then the integral equals the total magnetization current through the given area. Converting the integral into a contour integral of $\underline{M}$ around the area, it is seen that as long as no x dependence exists, the result vanishes.

Even if N does not depend on x, however, a weak dependence of B_z on x (as was assumed here) causes a non-vanishing total magnetization current, since the contributions of the segments with $x = x_1$ and $x = x_2$ to the contour integral then no longer cancel each other. To the lowest order of approximation the result turns out to be proportional to $\partial B_z/\partial x$; in a similar way, the current conducted by particle drifts is also proportional to $\partial B_z/\partial x$, which suggests that in nearly drift-free geometries the contributions of both these mechanisms to $\underline{j}$ are roughly of the same order.

It is interesting to compare these results to the work of Bird and Beard [1972] who computed numerically the contributions to $\underline{j}$ from the curvature drift current, the gradient drift current and the magnetization current, in a model field representing the magnetospheric tail (Figure 1 in their paper). As expected the first two sources of current tend to have opposite effects; the magnetization current has an appreciable effect on the dependence of $\underline{j}$ on z , but its contribution to the total current is small and the total current density from all sources is likewise small.

One also should be careful here to note (as was pointed out by one of the referees) that the guiding center approximation cannot always be assumed to hold. The mean energy of plasma sheet protons near $z = 0$ at $18\ R_e$ peaks between 2 and 5 kev [Bame et al., 1967] . Taking 3 kev as a typical value of the energy at $30\ R_e$ where $B_z \approx 2\gamma$, we find a gyration radius of about $0.6\ R_e$. On the other hand, if $\partial B_x/\partial z \approx 4\ \gamma/R_e$, the scale of field variation is about

$$B_z \,/\, (\partial B_x/\partial z) \approx 0.5\ R_e$$

This suggests that near the mid-plane of the plasma sheet many of the protons may be moving in non-adiabatic orbits. The preceding theory suggests that the motion of such protons, too, is nearly drift-free, but for more quantitative conclusions their orbits must be examined numerically, as has been done in the work of Pudovkin and Tsyganenko [1973]. Numerical work can also clarify the transition from the drift-free mode to the meandering mode of Sonnerup [1971] which, as was noted, is not drift free.

Appendix

For completeness, the vanishing of the mean drift velocity in the field of eq. (1) is derived here by straightforward integration. The gradient and curvature drifts are

$$\underline{v}_g = (v_\perp^2/2\omega B^2)\, \underline{B} \times \nabla B = -(v_\perp^2/2\omega)(B_x^2/B^3)\, dB_x/dz\, \hat{\underline{y}} \qquad \text{(A-1)}$$

$$\underline{v}_c = -(v_\parallel^2/\omega B^2)\ \underline{B} \times \left[\underline{B} \times (\nabla \times \underline{B})\right]$$

$$= (v_\parallel^2/\omega)\ (B_z^2/B^3)\ dB_x/dz\ \hat{\underline{y}} \qquad \text{(A-2)}$$

Assuming that the particle oscillates between mirror points with a period τ , the mean drift velocity is

$$\langle \dot{y} \rangle = (4/\tau) \int (\dot{y})_{av}\, dt \qquad \text{(A-3)}$$

where the integration extends from $z = 0$ to the mirror point and $(\ )_{av}$ implies averaging over a gyration period. It will be found convenient to use as integration variable the field intensity B observed by the particle and to express eq. (A-3) as

$$\langle \dot{y} \rangle = (4/\tau) \int_{B_z}^{B_m} (\dot{y})_{av}\ (dB/dt)^{-1}\, dB \qquad \text{(A-4)}$$

To evaluate dB/dt two unit vectors are introduced, $\hat{\underline{B}}$ aligned with $\underline{B}$ and $\hat{\underline{N}}$ orthogonal to in in the (x, z) plane :

$$\hat{\underline{B}} = B^{-1}\, (B_x\, \hat{\underline{x}} + B_z\, \hat{\underline{z}}) \qquad \text{(A-5)}$$

$$\hat{\underline{N}} = B^{-1}\, (-B_z\, \hat{\underline{x}} + B_x\, \hat{\underline{z}}) \qquad \text{(A-6)}$$

This gives

$$\underline{\hat{z}} = B^{-1} (B_z \underline{\hat{B}} + B_x \underline{\hat{N}}) \tag{A-7}$$

Using the z coordinate to measure the particle's progress along its guiding field line one gets

$$dB/dt = (dB/dz)(dz/dt) = (B_x/B)\, dB_x/dz\, (\underline{v} \cdot \underline{\hat{z}}) \tag{A-8}$$

by (A-7)

$$\underline{v} \cdot \underline{\hat{z}} = B^{-1} (B_z v_{\parallel} + B_x v_n) \tag{A-9}$$

where v_n is the velocity component along $\underline{\hat{N}}$. Averaged over a guiding center gyration, to lowest order

$$(v_n)_{av} = 0 \tag{A-10}$$

and therefore

$$(dB/dt)_{av} = (v_{\parallel} B_x B_z/B^2)\, dB_x/dz \tag{A-11}$$

Also, if the rapid gyration around the guiding center is averaged out, then $(\dot{y})_{av}$ of equation (A-4) simply becomes the combined drift velocity contributed by (A-1) and (A-2)

$$(\dot{y})_{av} = dB_x/dz\, (2\omega B^3)^{-1} (2 B_z^2 v_{\parallel}^2 - B_x^2 v_{\perp}^2) \tag{A-12}$$

Denoting by B_m the field intensity at the mirror point, one finds from the conservation of the magnetic moment

$$v_{\perp}^2 = v^2 \, (B/B_m)$$
$$v_{\parallel}^2 = v^2 \left[1 - (B/B_m) \right] \qquad \text{(A-13)}$$

Finally, if (A-11) to (A-13) are substituted in (A-4), the resulting expression has the form

$$\langle \dot{y} \rangle = \text{const.} \int_{B_z}^{B_m} \frac{2\, B_z^2\, B_m - B_z^2\, B - B^3}{(B^2 - B_z^2)^{1/2}\, (B_m - B)^{1/2}} \; \frac{dB}{B^2} \qquad \text{(A-14)}$$

The constant appearing here contains τ, ω/B, v and B_z; the derivative dB_x/dz cancels out while B_x itself is expressed in terms of B, as in the first factor in the denominator. It may then be shown that the integral vanishes, by dividing it into two parts and using integration by parts to show that they cancel. It follows that the mean guiding center drift vanishes, as has been claimed.

C a p t i o n s   t o   F i g u r e s

Figure 1 - Field lines in a drift-free magnetic field, with the projection of a trapped orbit schematically shown.

Figure 2 - Integration contour used in showing that in the absence of an x-dependence of plasma density and magnetic field, the contribution of the magnetization current to the total current in a drift-free model of the plasma sheet vanishes.

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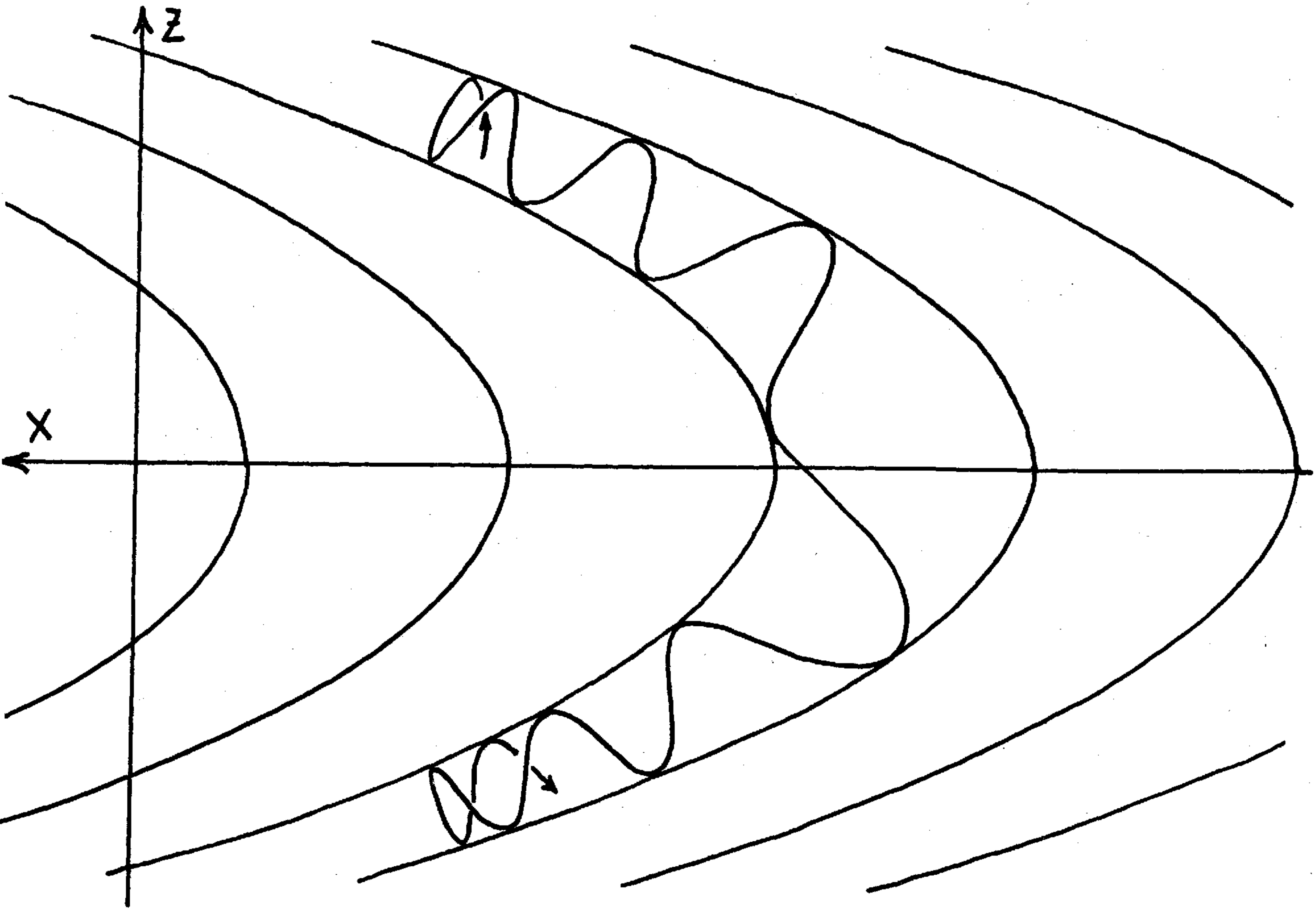

Figure 1

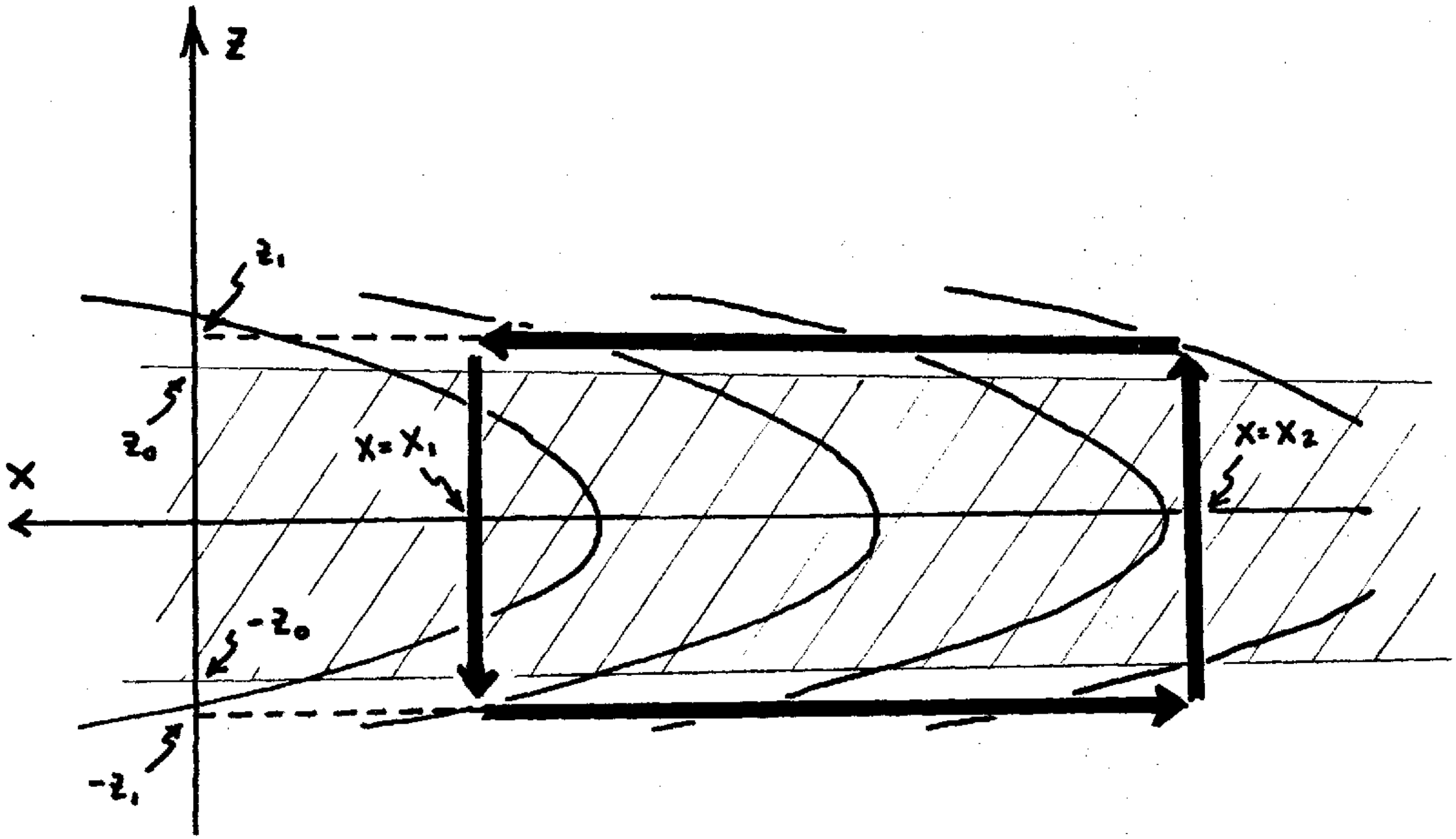

Figure 2